STUDY ON THE NUMERICAL ANALYSIS OF ROSSLER

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Abstract: Chaos is a robust phenomenon which appears in many branches of modern science. While the study of it is relatively young, many important results have been found which yield vast insight into nature of complex nonlinear dynamical systems. The techniques developed from these findings offer many exciting applications into future technology such as communication and neural networking.

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Introduction

The first person to notice some of the symptoms of chaos was the French mathematician *Henry Poincare* (1854-1912) in his studies of the gravitational three-body problem - the motion of three bodies (such as the sun, earth and moon) interacting via gravitational force. The equation of motion for this system is nonlinear and Poincare observed that tiny imprecision in the initial conditions would grow in time at an enormous rate. This extreme "sensitivity to initial conditions" mathematically present in the system studied by *Poincare* is called *"dynamical instability"*, or simply *"chaos"*.

Until recently, chaos was considered more of a nuisance than a valid scientific aspect. Whether it was turbulent fluid flow, fibrillation of the heart, the irregularity at which a tap dripped or the complexity of the weather, there was a little attempt to come to terms with phenomenon from a scientific point of view.

This changed when a meteorologist from MIT, USA named Edward Lorenz noticed some very interesting behavior in some equations he derived in an attempt to model the weather in 1961. He observed that in this set of nonlinear equations, making very small changes of the parameters had a huge effect on their solutions [3].

He coined the term *"butterfly-effect"* [4] to describe sensitive dependence of chaotic solutions. In his own words, it reads *"As small a perturbation as a butterfly fluttering its wings somewhere in the Amazons can in a few days time grow into a tornado in Texas"*. That is even a minute perturbation can cause realizable effects in a finite time under chaotic evolution of a system.

These results paved the way for a rigorous mathematical study of chaos. While there is no formal definition of the term, chaos can be most simply defined as the observable pattern of making a small change in a complex, nonlinear system which produces a huge change in the behavior of the system. This is often called a *sensitive dependence upon initial conditions* [4].

Chaos theory progressed rapidly after 20th century, when it became evident to some scientists that linear theory could not explain the observed behavior of certain experiments and were unable to predict such phenomenon. Usually it was considered that a nonlinear system cannot be predicted and analyzed but the situation is now quite different. The main catalyst for the development of chaos theory was electronic computers. Computers made repeated calculations to make the chaos a practical reality.

Now scientists have been recognizing the values of these studies as many subtle behaviors of physical systems have been shown to stem from a chaotic origin. As a result, there has been a tremendous increase of interest in chaotic behavior in such diverse fields as nuclear physics, biology, socioeconomics, electrical engineering and solid state physics.

Chaotic behavior is observed in natural systems, such as weather. This may be explained by analysis of a chaotic mathematical model which represents such a system. *Quantum chaos* investigates the relationship between *chaos* and *quantum mechanics*.

Chaotic behavior has been also observed in the laboratory in a variety of systems including electrical circuits, lasers, oscillating chemical reactions, fluid dynamics, and mechanical and magneto mechanical devices. Observation of chaotic behavior in nature include the dynamics of satellites in solar system, the time evolution of the magnetic field of celestial bodies, population growth in ecology, the dynamics of action potentials in neurons, and molecular vibrations. Everyday examples of chaotic systems include weather and climate [5]. There is some controversy over the existence of chaotic dynamics in plate tectonics and in economics [6-8].

THE NUMERICAL ANALYSIS OF ROSSLER

Numerical analysis can help to bring out a detailed picture of the dynamics of a nonlinear system such as Lorenz system, Rossler system, Duffing Oscillator etc. Many people studied chaos analytically, electronically but studying chaos numerically has its great importance[22].

Numerical study of chaos can be analyzed by the high speed digital computers and particularly, computer-driven graphics. These are the key tools that have made much of the progress in chaos and nonlinear dynamics possible. We need computers to generate the numerical solutions to nonlinear equations. Without some way of understanding that numerical output, however, little progress can be made. Computer graphics provide a way of visualizing the behavior of a nonlinear system and allowing us to build intuition about the solutions and how these change as parameters of the system change [2].

In order to study some basic aspects of the dynamics of Rossler system we numerically solve the Rossler equations:

$$\dot{x} = -(y+z), \tag{1.1}$$
$$\dot{y} = x + ay, \tag{1.2}$$
$$\dot{z} = b + z(x-c). \tag{1.3}$$

In numerical analysis, the Runge-Kutta Methods [16] are an important family of implicit and explicit iterative methods for the approximation of solutions of ordinary differential equations.

A popular choice of solving differential equation, by numerical techniques is the "4th order Runge-Kutta" scheme where successive estimates of the increment are made, and combined in a way that effectively includes higher order terms in the Taylor expansion.

Keeping in view the importance of Runge-Kutta Method, in the present study, we solve the Rossler system using it. The computer code is written in FORTRAN [17] and given at the end of this chapter.

1.1 NUMERICAL RESULTS OF ROSSLER SYSTEM

Since there are three control parameters a, b and c. In principle one can vary all three parameters simultaneously. But in order to avoid computational complexities in literature, it is suggested that fix any two parameters and study the dynamics as a function of the remaining parameter. Therefore, taking a guidance from literature, in the present study, the parameters a and b have been fixed and the dynamics of Rossler system is studied as a function of c. The values of x, y and z are computed as a function of time. Again taking help from the existing literature, plots of x and y are drawn for different values of a, b, and c and shown that how the dynamics of Rossler system evolves as c varies. The evolution of system is also explained through various x-t plots at different values of c.

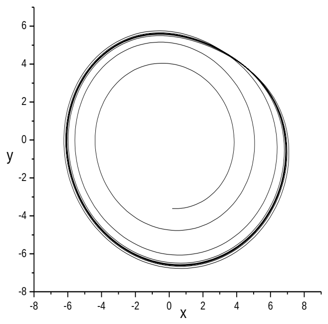

Fig.(1.1) Periodic-1 sol. a=b=0.1, c=4

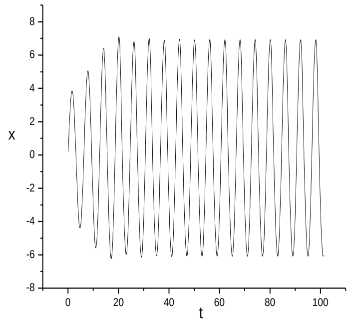

Fig.(1.2) Time series for period -1 a=b=0.1, c=4

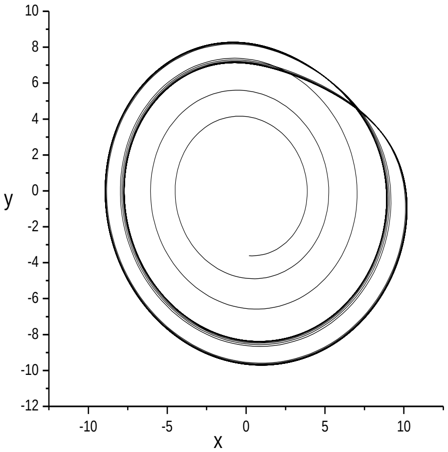

Fig.(1.3) Periodic-2 sol. a=b=0.1, c=6

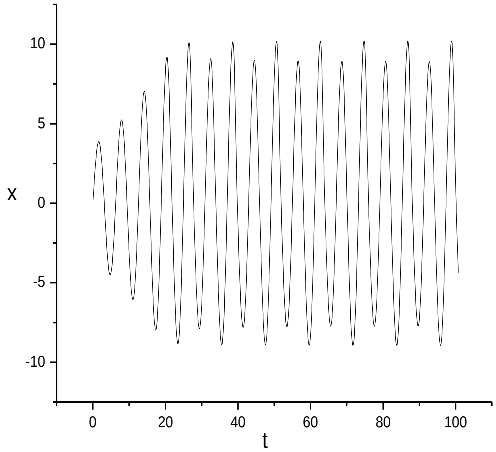

Fig.(1.4) Time series for period-2 a=b=0.1, c=6

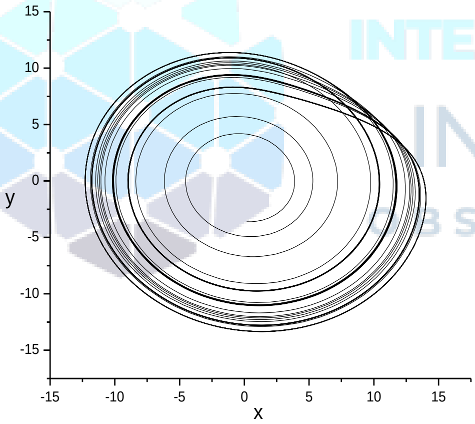

Fig.(1.5) Aperiodic sol. a=b=0.1, c=8.5

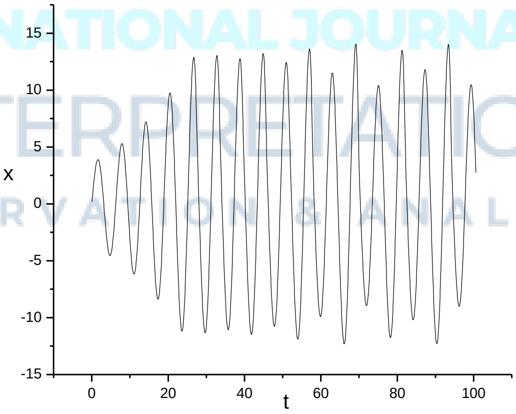

Fig.(1.6) Time series for Aperiodic sol. a=b=0.1, c=8.5

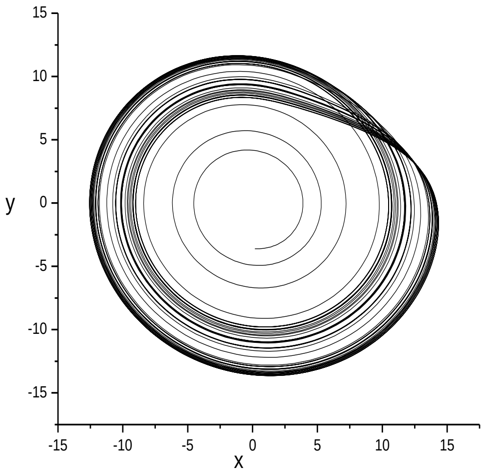

Fig.(1.7) Aperiodic sol. a=b=0.1, c=8.7

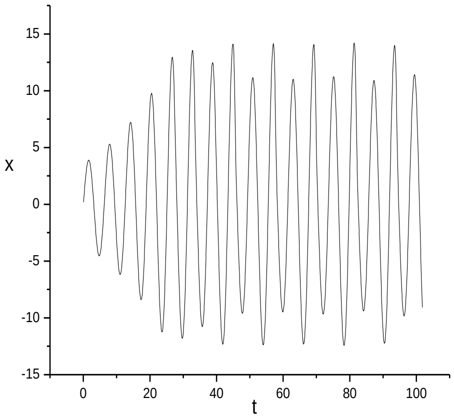

Fig.(1.8) Time series for Aperiodic sol. a=b=0.1, c=8.7

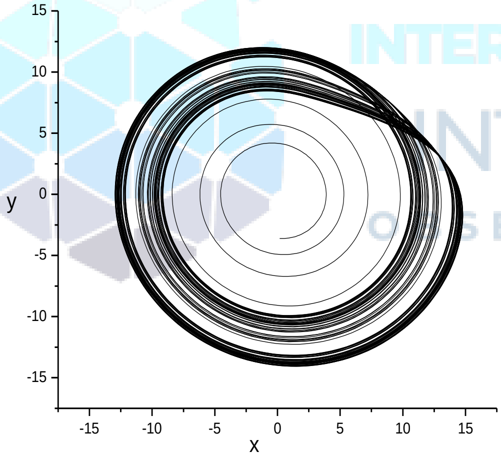

Fig.(1.9) Chaotic sol. a=b=0.1, c=9

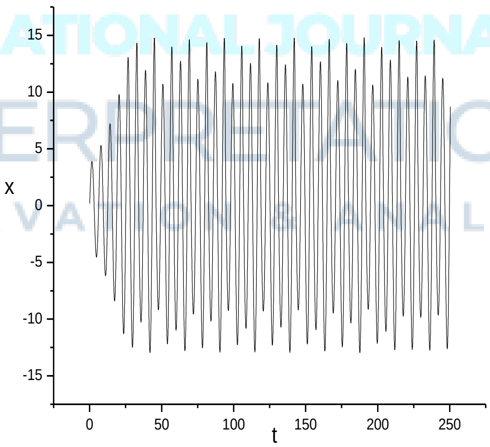

Fig.(1.10) Time series for Chaotic sol. a=b=0.1, c=9

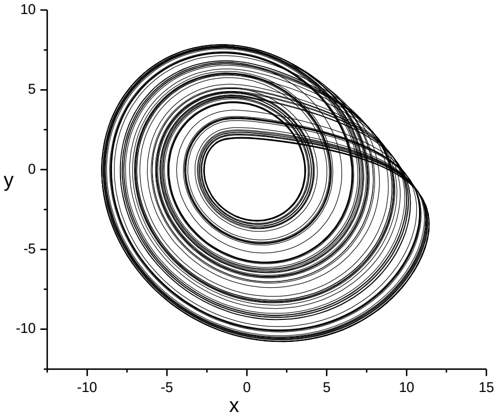

Fig.(1.11) Chaotic sol. a=b=0.2, c=5.7

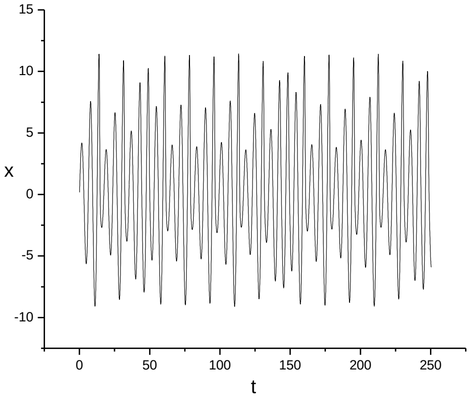

Fig.(1.12) Time series for Chaotic sol. a=b=0.2, c=5.7

For a=b=0.1 & c=4, we obtain Period -1 solution, which is shown in Fig.(1.1) and the corresponding time series plot is given by Fig.(1.2). When the parameter c becomes 6, the solution is again periodic but the time period doubled, Fig.(1.3). The time series of this case of drawn in Fig.(1.4). On further increase in c=8.5, the periodic solution break and aperiodic solution start appearing, this is depicted in Fig.(1.5) and (1.6). A further slight increase in c=8.7, gives rise same behavior as of the earlier one. This aperiodic dynamics is given in Fig.(1.7) and (1.8). If we choose c=9, the motion of the system becomes completely chaotic which is shown in Fig.(1.9) and (1.10).

It is also interesting to note that if we select a & b slightly more i.e. a=b=0.2 then even at smaller value of c=5.7, chaotic regime appear. This is shown in Fig. (1.11) and (1.12).

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