Fuzzy Edge Graceful Labeling on Double Wheel Graph

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Abstract: A graph G is said to be gracefully labeled if it admits a graceful numbering. A graph that allows a fuzzy graceful labeling is referred to as a fuzzy graceful graph. In this paper we introduce the notion of fuzzy edge graceful labeling for the double wheel graph and illustrate the concept through suitable example.

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Introduction: Fuzzy set theory provides a powerful mathematical framework for modeling uncertainty and imprecision inherent in many real-world phenomena. Since its introduction by Zadeh in 1965, the theory has evolved significantly, with foundational contributions by researchers such as Rosenfeld and Kaufmann, who extended fuzzy concepts to relations and graphs. A fuzzy graph, first defined by Kaufmann in 1973, offers a flexible representation of networks in which connections and membership values are not strictly binary but vary within a continuum. Graph labeling, another active area of graph theory, was initiated with the concept of graceful labeling introduced by Rosa in 1967. The fusion of fuzzy theory with graph labeling has led to the study of fuzzy graceful and related labeling schemes. In this paper, we explore **fuzzy edge graceful labeling** on the **double wheel graph**, a graph structure formed by joining a single hub vertex to two disjoint cycles of equal size. We define the notion of fuzzy edge graceful labeling in this context, establish conditions for its existence, construct explicit labeling schemes, and illustrate the results with examples. This study contributes to the growing body of work on fuzzy graph labeling and highlights the structural richness of the double wheel graph.

Preliminaries and Main Results

Definition: 1 [2]

Let U and V be two sets. A fuzzy relation ρ from U to V is defined as a fuzzy set of the cartesian product $U \times V$. A fuzzy graph $G = (\sigma, \mu)$ consist of two functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$. These functions must satisfy the condition thatfor all $u, v \in V$, we have $\mu(u,v) \leq min\{\sigma(u), \sigma(v)\}$.

Definition: 2 [4]

A labelling of a graph is an assignment of values to the vertices and edges of a graph.

Definition: 3 [3]

A graph $G = (\sigma, \mu)$ is called a fuzzy labeling graph if the following conditions are hold: $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ are bijective functions, the membership value of vertices and edges are distinct and for all $u, v \in V, \mu(u,v) < min\{\sigma(u), \sigma(v)\}$.

Definition:4 [4]

A graceful labelling of a graph G with q edges is an injection f: V(G) → {0,1, 2,….q} such that when each edge xy ϵ E(G) is assigned the label |f(x) – f(y)| , all of the edge labels are distinct.

Definition: 5 [1]

An edge graceful labeling is defined as, In a graph G, we denote the set of edges by $E(G)$ and the vertices by $V(G)$. Let q be the cardinality of $E(G)$ and p be that of $V(G)$. Once the labeling of edges is given, a vertex u of the graph is labelled by the sum of the labels of the edges incident to it, modulo p. ie, $V(u) = \sum E(e) mod |V(G)|$, where $V(u)$ is the label for the vertex and $E(e)$ is the assigned value of an edge incident to u.

A graph G is said to be Edge graceful if it admits edge graceful labeling.

Definition: 6 [4]

A Wheel graph Wn is a graph with n vertices (n≥ 4), formed by connecting a single vertex to all the vertices of an (n-1) cycle.

A Wheel graph with fuzzy labeling is called fuzzy wheel graph

Definition: 7 [4]

A Double Wheel graph DWn of order n can be composed to 2CN+K1. (ie)., it consists of order N, where the vertices of the two cycles are all connected to a common center.

A Double wheel graph with fuzzy labeling is called a fuzzy Double wheel graph.

Definition: 8

In a fuzzy double wheel graph DW_n if all the edge values are distinct then it is called a fuzzy edge graceful double wheel graph.

Proposition: 9

For some $n \geq 4$ the double wheel graph DW_n is fuzzy edge graceful double wheel graph where edges connecting central vertex (v_0) to inner cycle and outer cycle are $\mu(v_0, w_i) = (2i-1)\,C$ and $\mu(v_0, u_i) = 2iC$ respectively and $\mu(u_i, u_{i+1}) = (2n+i)\,C$, $\mu(u_n, u_1) = 3nC$ for inner cycle and $\mu(w_i, w_{i+1}) = (3n+i)\,C$, $\mu(w_n, w) = 4nC$ for outer cycle and C can be choose $C = 0.01$

Proof

A Double wheel graph DW_n is a graph with $2n+1$ vertices and $4n$ edges exists only if

$n \geq 4$

In a double wheel graph v_0 is the central vertex, w_i denote the vertices in the inner cycle, u_i denote the vertices in the outer cycle such that $\mu(v_0, w_i) > 0$, $\mu(v_0, u_i) > 0$, $\mu(w_i, w_{i+1}) > 0$ and $\mu(u_i, u_{i+1}) > 0$ where $i = 1,2 \ldots n$ with $w_{n+1} = w_1$ and $u_{n+1} = u_1$

Here $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ defined by

$\mu(v_0, w_i) = (2i-1)\,C$

$\mu(v_0, u_i) = 2iC$ where $i = 1,2 \ldots n$

$\mu(u_i, u_{i+1}) = (2n+i)\,C$

$\mu(u_n, u_1) = 3nC$

$\mu(w_i, w_{i+1}) = (3n+i)\,C$

$\mu(w_n, w) = 4nC$, where $i = 1,2 \ldots n-1$.

$\sigma(v_0) = \sum_{i=1}^{n} \mu(v_0, w_i) + \sum_{i=1}^{n} \mu(v_0, u_i)$

$= C\,[1 + 3 + \cdots \ldots \ldots (2n-1)] + C\,[2 + 4 + \cdots \ldots + 2n]$

$= n^2 C + C\,(n^2 + n)$

$= C\,(2n^2 + n)$

$\sigma(w_1) = \mu(v_0, w_1) + \mu(w_n, w_1) + \mu(w_1, w_2)$

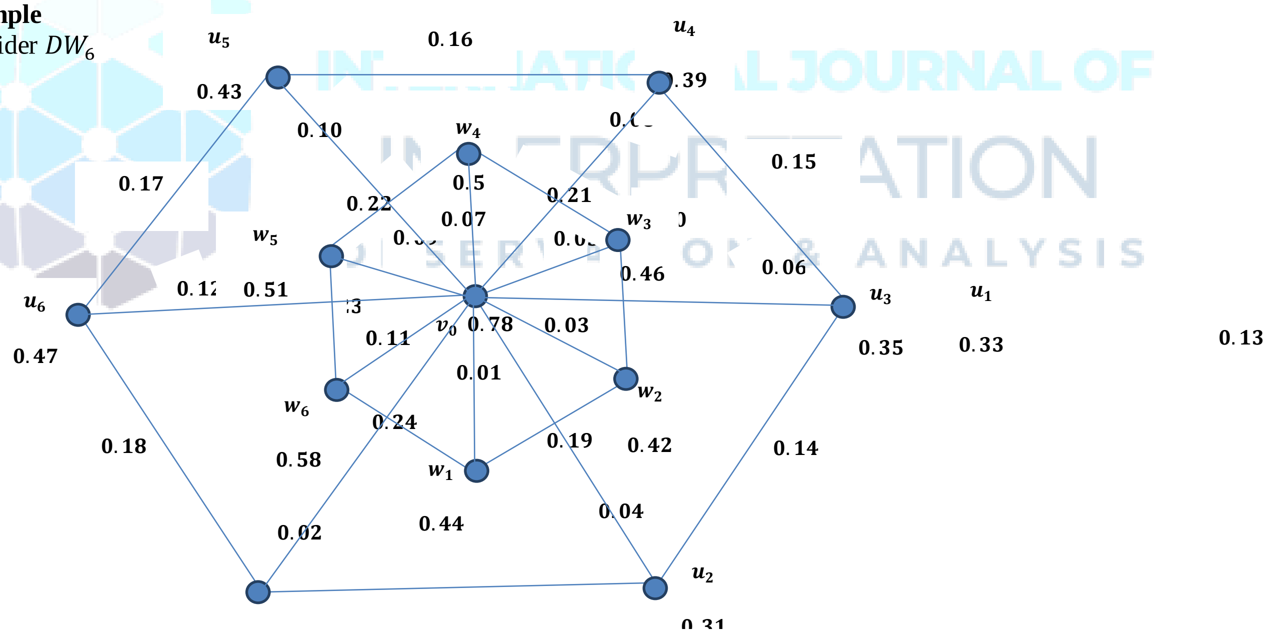
$\sigma(w_i) = \mu(v_0, w_i) + \mu(w_{i-1}, w_i) + \mu(w_i, w_{i+1})$ where $i = 2,3 \ldots n$

$\sigma(u_1) = \mu(v_0, u_1) + \mu(u_n, u_1) + \mu(u_1, u_2)$

$\sigma(u_i) = \mu(v_0, u_i) + \mu(u_{i-1}, u_i) + \mu(u_i, u_{i+1})$ where $i = 2,3 \ldots n$

Example

Consider DW_6



Conclusion

In this paper, we examined the concept of fuzzy edge graceful labeling for the double wheel graph. The results presented here lay the groundwork for extending this investigation to additional families of special graphs in future studies.

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