

Polynomial Functions and its Common Core State Standards

Balbinder Singh

Research Scholar, Department of Mathematics, SunRise University, Alwar, Rajasthan (India)

Corresponding Author Email: singhbalbinder1986@gmail.com

Abstract: Polynomial functions and their properties are fundamental to algebra, calculus, and mathematical modeling. Students who do not have a strong understanding of the relationship between factoring and solving equations can have difficulty with optimization problems in calculus and solving application problems in any field. Understanding function transformations is important in trigonometry, the idea of the general antiderivative, and describing the geometry of a problem mathematically. This paper presents spreadsheet activities designed to bolster students' conceptualization of the factorization theorem for polynomials, complex zeros of polynomials, and function transformations. These activities were designed to use a constructivist approach involving student experimentation and conjectures.

[Singh, B. **Polynomial Functions and its Common Core State Standards**. *The International Journal of Interpretation, Observation and Analysis*, 2026; Volume 1, Issue 1 (January-March): Page Number: 77-81. ISSN 2349-0713, Peer-reviewed (online/offline), Refereed, Indexed and International Journal (Since 2013), Global Impact Factor: 6.489

Keywords: Experimentation with functions, function transformations, polynomial theory, factorization theorem for polynomials, conjugate pairs theorem, spreadsheets, active learning, inquiry-based learning

Introduction: Through their study of quadratic functions, students have begun to recognize properties of the graph of a function such as the extrema, x- and y-intercepts, symmetry of the graph with respect to a line, concavity, and end behavior. The study of polynomial functions extends the study of non-linear functions started with quadratic functions to include functions of the form $y = P(x)$ where $P(x)$ is a polynomial of degree $n \geq 2$.

Mastery of the unit will require students to develop their understanding of: 1) the relationship between zeros of the polynomial function and the factors of the related polynomial; 2) the x- and y-intercepts of the function; 3) the end behavior of the y-values of the function as x approaches positive or negative infinity; 4) the transformations of the graph of $y=f(x)$ when it is reflected over the x- and y-axes and under what conditions will the function be odd or even, i.e. when $f(-x) = -f(x)$ or when $f(-x) = f(x)$; 5) relative extrema of polynomial functions and how to interpret such values in the context of real-world problems; 6) operations involving polynomial expressions, including division of polynomials and the connection of division to the Division Algorithm, the Remainder Theorem, and the Factor Theorem; 7) factoring polynomials by various methods including the application of the Remainder Theorem and advanced polynomial identities; 8) the connection between binomial expansion and Pascal's Triangle; 9) applications of polynomials in modeling real-world functions, including solving counting problems by using generating functions; and 10) the difference

between exponential growth versus growth modeled by polynomial functions.

The unit includes 6 investigations through which the topics will be developed. Each investigation has several activities that develop the objectives of the unit. The activities are designed to allow students to explore relationships and formulate conjectures related to the basic principles in working with polynomials. Technology is used in many activities to allow students to manipulate parameters in a function in order to discover patterns in the behavior of the graph of the function. Most activities are written to enable students to do use any graphing utility available in their classroom, but many activities were written using GeoGebra since the algebraic, graphic, and numerical representations of the polynomial functions are readily available.

Investigation 1 examines basic graphs of polynomial functions. Included in this investigation are activities to develop: a) the relationship between zeros of the polynomial and the factors of the polynomial; b) the x- and y-intercepts of the function; c) the end behavior of the y-values of the function as x approaches positive or negative infinity; d) odd and even functions; e) the impact of the multiplicity of a linear factor of a polynomial function on the graph of the function; and f) extrema of polynomial functions and how to interpret such values in the context of real-world problems.

Investigation 2 examines polynomial operations and their connection to the algebraic structure formed by integers under addition and multiplication. Similar properties between the two structures used include closure of addition, subtraction, and multiplication of polynomials and the zero product property. The activities start with an application in geometry that has students calculate the surface area and volume of a rectangular solid. Subsequent activities study the process of long division and the connection between the Remainder Theorem and the Factor Theorem. Students should see this as an extension of the division algorithm, in that, when the remainder of the division is zero, the divisor is a factor of the polynomial. Students will see that knowing that if $P(a)=0$ for a polynomial, $P(x)$, then $P(x)$ can be factored where one factor is $(x-a)$ and the other factor is the quotient $P(x) \div (x-a)$.

Investigation 3 examines the process of factoring polynomials. Techniques for factoring polynomials are based on the connection between the zeros of the polynomial, the x-intercepts of the graph of the polynomial function, and the factors of the polynomial as established by the Factor Theorem. When extended to the complex numbers, it can be shown that every polynomial has at least one linear factor with complex coefficients, i.e., that the Fundamental Theorem of Algebra is established. Note that having one factor leads to the conclusion that every polynomial can be completely factored into linear factors over the complex numbers. Polynomials with complex zeros and real coefficients can be factored into quadratic factors that are the product of linear factors of complex conjugate zeros. In addition, identities established in previous units for quadratic expressions can be applied to factor polynomials of higher degree such as $ax^4 + bx^2 + c$, $x^n - y^n$, or $x^n + y^n$. Other special identities, such as the sum of the positive integers $1+2+3+\dots+n$ and the sum of squares of the first n positive integers, are presented as enrichment activities for advanced students.

Investigation 4 has students look for patterns in the expansion of $(a+b)^n$. In order to set up this investigation, students will first examine the patterns in Pascal's triangle. To develop the triangle, students will study the subsets of sets with 0, 1, 2, 3, 4, and 5 elements. This can be done in the context of the number of different pizzas possible with 0, 1, 2, 3, 4, or 5 possible toppings. Other appropriate contexts such as the number of subcommittees that can be formed from a given committee could be used in the discussion. Once the connection between the number of subsets of a set of n elements and the numbers in

Pascal's Triangle is established, students can then examine binomial expansions. By examining the connection between the number of subsets with k elements of a set with n elements and the coefficient of the term $a^{n-k} \cdot b^k$ in the expansion of $(a+b)^n$, students will understand from where the coefficients of the binomial expansion come. The Binomial Theorem will be used to expand the power of any binomial.

Investigation 5 examines the application of polynomial functions and expressions in solving problems. Two investigations are designed to use students' knowledge of polynomial functions to solve problems that can be modeled by polynomial functions. Students will answer questions related to properties of the graph of the function such as x-intercepts, y-intercepts, intervals where the function is increasing or decreasing, and relative maxima and minima of the function. In the first activity, students model data involving revenue from a software company and the second model has students attempt to create a polynomial model for a roller coaster ride. A final activity extends the properties of results of binomial expansion to generating functions. This investigation will likely be attempted by classes that are seeking ways to enrich the course and that have enough time to do so. The study of generating functions is an advanced topic in discrete mathematics. One of its applications uses the coefficients of the product of two or more polynomials to solve counting problems. For example, suppose you wanted to interview six people from a group of 20 people that are separated into four categories with indistinguishable members, with 4 in one category, 6 in another, 7 in another, and 3 in the last category, with a minimum of 1 person from each category. This coefficient of the term x^6 of the product $(x + x^2 + x^3 + x^4 + x^5 + x^6)(x + x^2 + x^3 + x^4)(x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7)(x + x^2 + x^3)$ would say how many different interviews could be conducted. Additionally, suppose you wanted to see how many ways 10 coins made up of a combination of pennies, dimes, or quarters can total up to a value of \$1. You can take the trinomial $(x + x^{10} + x^{25})$ and raise it to the 10^{th} power. The coefficients of the resulting polynomial are the number of ways that you could pick 10 coins to add up to the amount of money equaling the exponent of the term. Therefore, the coefficient of the term x^{100} , if that term exists in the expansion, is the number of ways to make \$1 from the 10 coins. While this expansion would be cumbersome doing computation by hand, it is accessible using a computer algebra system like the

one available with the GeoGebra software or on a TI-NSpire.

Investigation 6 compares growth that can be modeled by polynomial equations versus exponential functions. The unit is launched with an investigation involving a computer science application about the number of resistors on an integrated panel that doubles every two years. The remaining two investigations seek to demonstrate that if the base of an exponential function is greater than 1, the graph of the equation modeling exponential growth will always eventually be greater than the graph of the equation modeling the growth of any polynomial function, regardless of its degree. Activity 3.6.3 would likely be addressed by advanced classes only in order to provide a more formal explanation of why exponential growth with a base greater than 1 will eventually outgrow any polynomial growth.

Essential Questions:

- What are the basic features of a polynomial function based on the degree of the polynomial?
- How is the division of a polynomial, $P(x)$, by a binomial of the form $x - a$ connected to the polynomial function, $y = P(x)$, when evaluated at $x=a$?
- How can long division by the binomial $x-a$ be used to factor a polynomial?
- What is the connection between the zeros of a polynomial function, the x -intercepts of the graph of the polynomial function, and the factors of the polynomial?
- How is the expansion of the binomial $(a+b)^n$ connected to Pascal's Triangle?
- How can polynomial functions be applied to applications in combinatorics and mathematical models?
- How does polynomial growth compare to exponential growth?

Enduring Understandings:

- Polynomial functions of odd degree will have opposite end behavior as x approaches $+$ or $-\infty$, while polynomial functions of even degree will have the same end behavior as x approaches $+$ or $-\infty$.
- The x -intercepts of a the graph of a polynomial function correspond to at least one of the factors of the polynomial.
- Exponential growth with a base greater than 1 will always eventually exceed polynomial growth as x approaches $+\infty$.

Common Core State Standards

- F.IF.7c Graph polynomial functions,

identifying zeros when suitable factorizations are available, and showing end behavior.

- F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; multiplicity of roots; symmetries; end behavior; and periodicity.
- A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2) \cdot (x^2 + y^2)$.
- A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples
- A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)
- A.CED.2 Create equations in two or more variables to represent relationships between

quantities; graph equations on coordinate axes with labels and scales.

- F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) For example given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Unit Contents:

Investigation 1: Basic Graphs of Polynomial Functions (6 days)

Investigation 2: Operations of Polynomials (4 days)

Investigation 3: Factoring Polynomials Using the Factor Theorem and Identities (5-6 days)

Investigation 4: Binomial Theorem (3 days)

Investigation 5: Applications of Polynomial Functions (2-3 days)

Investigation 6: Exponential Growth versus Polynomial Growth (2-3 days)

Performance Task: Creating Art via Polynomials (1 day for presentations)

Mid-unit Assessment (1 day)

Review for Unit Assessment (1 day)

Unit Assessment (1 day)

Vocabulary

Algebraic Identity
Array
Binomial
Binomial Theorem
Coefficient/Leading Coefficient
Combinations
Complex Coefficient
Compound interest
Decreasing Function
Degree
Division Algorithm
Divisor
End behavior
Even Function
Exponential Function
Exponential Growth
Extrema
Factor Theorem
Generating Polynomial
Generating Function
Increasing Function

Assessment Strategies:

Performance Task

In the Unit 3 performance task, students will apply their knowledge of polynomial functions and their related graphs to replicate an image from art or from the real world using polynomial equations over a restricted domain in a graphing utility. The choice of the image is up to the student and the choice of the graphing utility will depend on its availability to the students. It is expected that all students will have access to a TI-83/84 version of a graphing calculator in their schools. Another method would be through the use of online graphing utilities like GeoGebra and Desmos. If the teacher wants all the students doing the same image, a copy of a GeoGebra file with an image of a sculpture on the Southern Connecticut State University campus is provided. It includes many curves that can be modeled by linear, quadratic, cubic, or higher order polynomials. Instructions on how to restrict the domain are given as a resource.

Other Evidence (Formative and Summative Assessments)

- Exit Slips
- Classwork
- Homework Assignments
- Math Journals
- Unit 3 Mid-unit Assessment
- Unit 3 End of Unit Assessment

Pascal's Triangle
Polynomial
monomial
binomial
trinomial
cubic
quartic
Power
Quadratic Factor
Quotient
Reflection
Regression Equations
Remainder
Remainder Theorem
Root
Scatter plots
Series
Subsets
Transformation x-Intercepts
y-Intercepts
Zero

Linear Factor Mathematical Model Maximum Minimum Odd Function	Zero of multiplicity Zero Product Property
---	---

References:

- [1] Alagic, M., & Palenz, D. (2006). Teachers explore linear and exponential growth: Spreadsheets as cognitive tools. *Journal of Technology and Teacher Education*, 14(3), 633-649.
- [2] Clarke, T., Ayres, P., & Sweller, J. (2005). The impact of sequencing and prior knowledge on learning mathematics through spreadsheet applications. *Educational Technology Research and Development*, 53(3): 15-24.
- [3] Common Core State Standards Initiative (2013). Common Core State Standards for Mathematics. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf
- [4] Grady, M., Watkins, S., & Montalvo, G. (2012). The effect of constructivist mathematics on achievement in rural schools. *The Rural Educator*, 33(3): 38-47.
- [5] Hsiao, F. S. (1985). Micros in mathematics education - Uses of spread- sheets in CAL. *International Journal of Mathematical Education in Science and Technology*, 16(6): 705-713.
- [6] Microsoft Support (2010) Floating-point arithmetic may give inaccurate results in Excel. Retrieved from <https://support.microsoft.com/en-us/kb/78113>
- [7] Ratti, J. S. & McWaters, M. (2015). College algebra and trigonometry(3rd ed.). New York, NY: Pearson.
- [8] Rau, J. V. (1994). Constructivism, factoring, and beliefs. *School Science and Mathematics*, 94(8): 421-426.
- [9] Sadri, P. (2015). Spreadsheets: Laying a foundation for understanding functions. *Spreadsheets in Education*. 8(2).

