STUDY ON CHAOS IN ROSSLER SYSTEM

Dr. Shweta
Guest Lecturer, Government First Grade College for women, Raichur, Karnataka (India)
Email: sanju134589@gmail.com

Abstract: Chaos theory is based on simple deterministic systems that demonstrate random behavior. In a chaotic system, even tiny changes in initial conditions eventually lead to major changes in state. This is called "sensitive dependence on initial conditions". This feature of chaotic systems is used in this for improving the security of analog transmission. The seminar consists of parts, which are: - general review - orbit diagram - time series, phase trajectory, power spectra and Poincare map analysis for selected values of parameter c - the attractor reconstruction, estimation of correlation dimension and compatison of the reconstructed system with the original attractor

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Introduction: Rossler systems were introduced in the 1970s as prototype equations with minimum ingredients for continuous-time chaos. Since the Poincare-Bendixson theorem precludes the existence of other than steady, periodic, or quasiperiodic attractors in autonomous systems defined in one or two dimensional manifolds such as line, the circle, the plane, the sphere, or the torus [9], the minimal dimension for chaos is three.

On this basis Otto Rossler came up with a series of prototype systems of ordinary differential equations in 3-dimension phase space [10-12]. He also proposed 4-dimension systems for hyperchaos that is chaos with more than one positive Lyapunov exponent [12-13].

Rossler was inspired by the geometry of flows in dimension three and in particular, by the reinjection principle, which is based on the feature of relaxation type systems to often present a z-shaped slow manifold in their phase space. On this manifold, the motion is slow until an edge is reached whereupon the trajectory jumps to the other branch of the manifold, allowing not only for periodic relaxation oscillations in dimension two, but also for higher types of relaxation behavior as noted by Rossler [12]. In dimension three, the reinjection can induce chaotic behavior if the motion is spiraling out on one branch of the slow manifold. In this way, Rossler invented a series of systems, the most famous of which is a simplified version of Lorenz equations and given as

$$\frac{dx}{dt} = -y - z, \tag{1.1}$$

$$\frac{dy}{dt} = x + ay, \tag{1.2}$$

$$\frac{dz}{dt} = b + z(x - c). \tag{1.3}$$

Where a, b and c are parameters while x, y and z are dynamical variables.

Rossler argued on the basis of his reinterpretation of the Schwarzschild metric that the Black Holes do not emit Hawking radiation and will thus exist forever.

Rossler varied the values of parameter c by fixing the values of other two parameters a, b same. He shows that how attractor has been changing with the variation of c. Same things he did by constructing electronic circuit. Both were same and he concluded that with the variation of c largely chaos can be seen. The circuit constructed by Rossler [14] is given in the Fig. (1.1).

Rossler studied the chaotic attractor with a=0.2, b=0.2, and c=5.7, though properties of a=0.1, b=0.1 and c=14 have been more commonly used since.

Nonlinear term is present in the last equation i.e. xz. By fixing two of parameters and varying the third one can study approach to chaos.

1.1 VARIATION OF PARAMETERS: BIFURCATION DIAGRAM

Some properties of the Rossler system can be deduced via linear methods such as Eigen vectors, bur main features of the system require nonlinear methods such as bifurcation diagram.

Rossler attractor's [10] behavior is largely a factor of the value of its constant parameter (a, b and c). In general varying each parameter has a

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comparable effect by causing the system to converge toward a periodic orbit, fixed point, or escape toward infinity; however the specific ranges and behaviors induced vary substantially for each parameter. Periodic orbits, or "unit cycles," of the Rossler system are defined by the number of loops around the central point that occur before the loops series begins to repeat itself.

Bifurcation diagrams [1-2] are a common tool for analyzing the behavior of chaotic systems. Bifurcation diagrams for the Rossler attractor are created by iterating through the Rossler ODEs holding two of the parameters constant while conducting a parameter sweep over a range of possible values for the third.

1.1.1 Variation of b

The effect of b on the Rossler attractor's behavior is best illustrated through a bifurcation diagram. This bifurcation diagram was created with a=0.2, c=5.7.

As shown in the accompanying Fig. (1.2) diagram, as b approaches 0 the attractor approaches infinity (note the upswing for very small values of b). Comparative to the other parameters, varying b seems to generate a greater range when period-3 and period-6 orbits will occur. In contrast to a and c, higher values of b systems that converge on a period-1 orbit instead of higher level orbits or chaotic attractors.

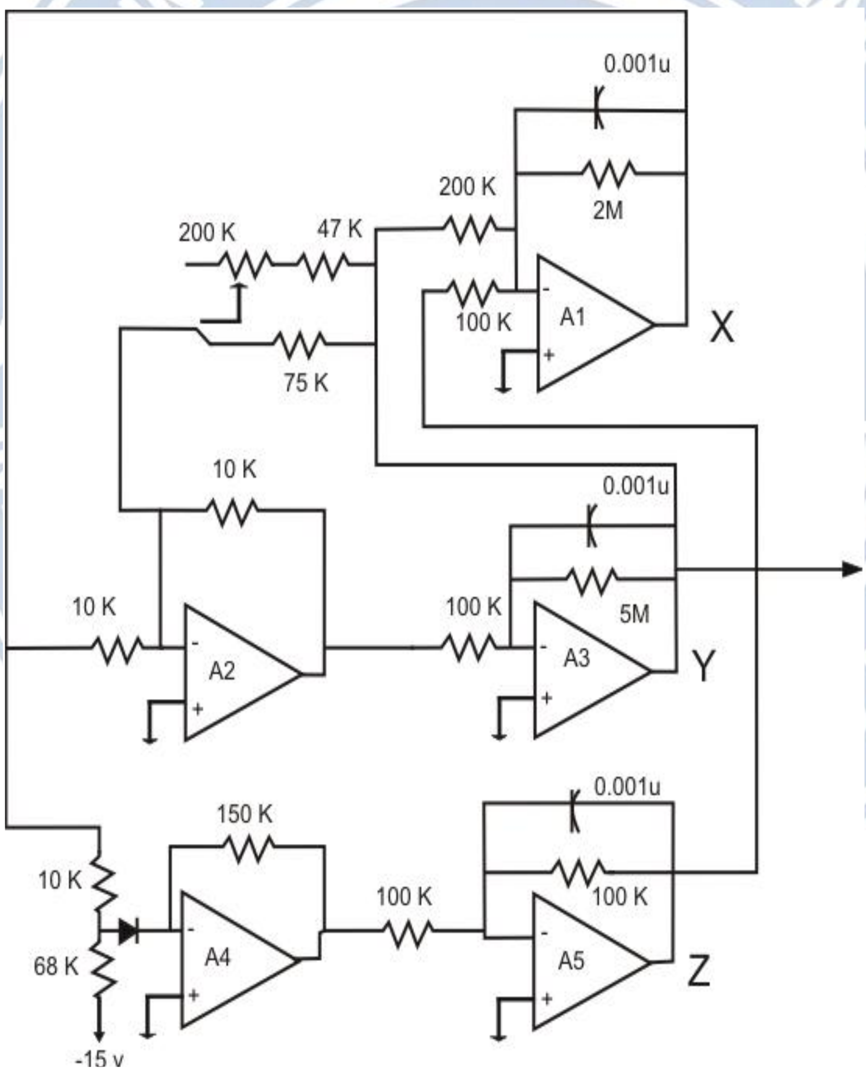

Fig. (1.1) Schematic circuit of Rossler drive

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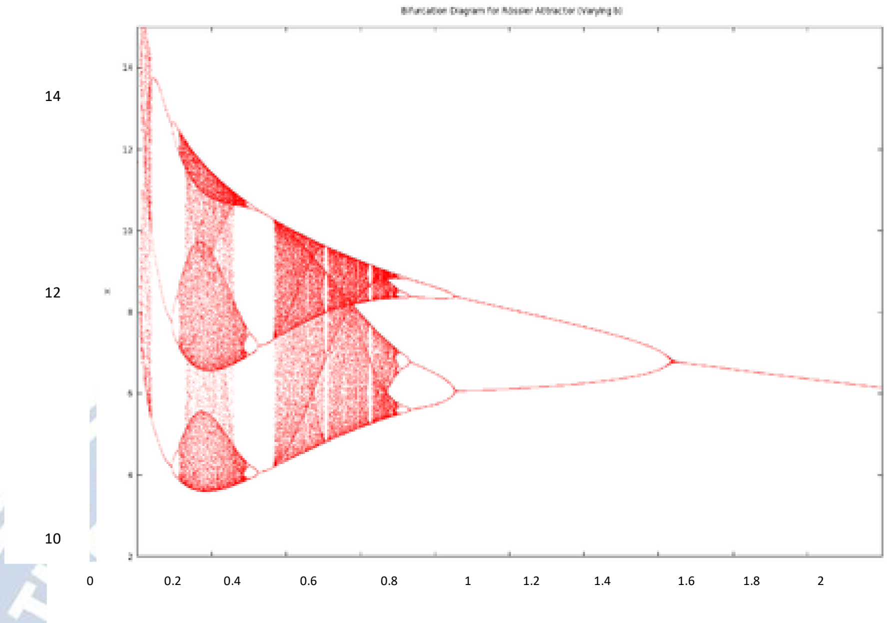

Fig. (1.2) Bifurcation diagram for the Rossler attractor for varying b

1.1.2 Variation of c

The traditional bifurcation diagram for Rossler attractor is created by varying c with a=b= 0.1. This bifurcation diagram reveals that low values of c are periodic, but quickly become chaotic as c increases. This pattern repeats itself as c increases there are sections of periodicity interspersed with periods of chaos, although the trend is towards higher order periodic orbits in the periodic sections as c increases. For example, the period one orbit appears for values of c around 4 and is never found again in the bifurcation diagram. The same phenomena is seen with period three; until c=12, period three orbits can be found, but thereafter, they do not appear.

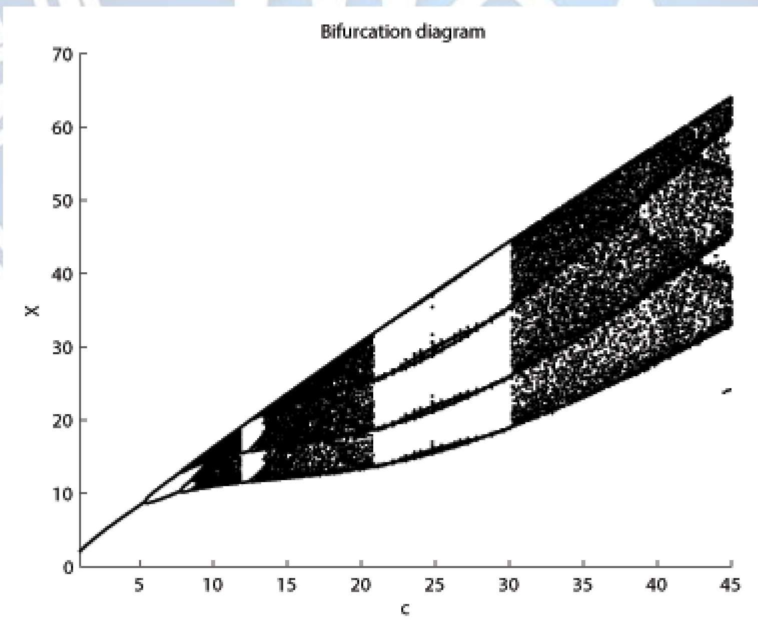

Fig. (1.3) Bifurcation diagram for the Rossler attractor for varying c

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A graphical illustration of the changing attractor over a range of c values illustrates the general behavior seen for all of these parameter analyses-the frequent transitions from ranges of relative stability and periodicity to complete chaotic and back again.

1.2 LINEAR STABILITY ANALYSIS OF ROSSLER SYSTEM

It is well known that there are no general methods to solve nonlinear systems. However, even when the problem is not exactly solvable, we can start the analysis from the simplest possible states or structures admitted by the system, which we may be able to identify by inspection or by simple analysis.
In the same spirit, here we find the stability of fixed points [1] for Rossler system.

1.1.1 Fixed Points

One obvious physically important and relevant state of the system is the *equilibrium* or *fixed points* of the system. Any admissible solution of $F(X) = 0$, which we call as X^*, then gives as *equilibrium* or *fixed point* of the system. This is because if X^* is a solution of $\dot{X} = 0 = F(X)$ at a given time, it continues to be so for all times, and so it is a fixed *point*.

In order to find the fixed points, the three Rossler equations set to zero and the (x,y,z) coordinates of each fixed point were determined by solving the resulting equations. This yields two fixed points in space as

$$(x^*, y^*, z^*) = \left(\frac{c\pm\sqrt{c^2-4ab}}{2}, \frac{-c\pm\sqrt{c^2-4ab}}{2a}, \frac{c\pm\sqrt{c^2-4ab}}{2a}\right), \qquad (1.4)$$

which in turn can be used to show the actual *fixed points* for a given set of parameter values.

1.1.2 Eigen Values

Stability of equilibrium points can be easily analyzed by finding the Jacobian of Rossler system's equation.

The Jacobian [15] of a function describes the orientation of a tangent plane to the function at a given point. In this way, the Jacobian generalizes the gradient of a scalar-valued function of multiple variables which itself generalizes the derivative of a scalar-valued function of a scalar.

The importance of the Jacobian lies in the fact that it represents the best linear approximation to a differentiable function near a given point.

$$\det(M - \lambda I) = \begin{vmatrix} -\lambda & -1 & -1 \\ 1 & a-\lambda & 0 \\ z & 0 & x-c-\lambda \end{vmatrix} = 0$$

Expanding the determinant, we obtain

$$-\boldsymbol{\lambda}^3 + \boldsymbol{\lambda}^2 A + \boldsymbol{\lambda} B + C = 0 \qquad (1.5)$$

Where $A = a + x^* - c$

$B = ac - ax^* - 1 - z^*$

$C = x^* - c + a z^*$

Note that in the calculation of A, B and C we have used x^*& z^* from first set of *fixed points* given in equation (1.4). The Eigen values of equation (1.5) are computed using MAPLE 6 and given as.

$$\lambda_1 = \frac{1}{6}\left(36\,BA + 108C + 8A^3 + 12\sqrt{-12B^3 - 3B^2A^2 + 54ABC + 81C^2 + 12CA^3}\right)^{1/3} + \frac{6\left(-\frac{1}{3}B + \frac{1}{9}A^2\right)}{\left(36BA + 108C + 8A^3 + 12\sqrt{-12B^3 - 3B^2A^2 + 54BAC + 81C^2 + 12CA^3}\right)^{1/3}} + \frac{1}{3}A \qquad (1.6)$$

$$\lambda_2 = -\frac{1}{12}\left(36\,BA + 108C + 8A^3 + 12\sqrt{-12B^3 - 3B^2A^2 + 54ABC + 81C^2 + 12CA^3}\right)^{1/3} + \frac{3\left(-\frac{1}{3}B + \frac{1}{9}A^2\right)}{\left(36BA + 108C + 8A^3 + 12\sqrt{-12B^3 - 3B^2A^2 + 54BAC + 81C^2 + 12CA^3}\right)^{1/3}} + \frac{1}{3}A +$$

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$$\frac{1}{2}I\sqrt{3}\left(\frac{1}{6}\left(36BA+108C+8A^3+12\sqrt{-12B^3-3B^2A^2+54BAC+81C^2+12CA^3}\right)^{1/3}+\frac{6\left(-\frac{1}{3}B+\frac{1}{9}A^2\right)}{\left(36BA+108C+8A^3+12\sqrt{-12B^3-3B^2A^2+54BAC+81C^2+12CA^3}\right)^{1/3}}\right) \quad (1.7)$$

$$\lambda_3 = \frac{3\left(-\frac{1}{3}B+\frac{1}{9}A^2\right)}{\left(36BA+108C+8A^3+12\sqrt{-12B^3-3B^2A^2+54BAC+81C^2+12CA^3}\right)^{1/3}}+\frac{1}{3}A-\frac{1}{2}I\sqrt{3}\left(\frac{1}{6}\left(36BA+108C+8A^3+\right.\right.$$

12−12B3−3B2A2+54BAC+81C2+12CA3

13+6−13B+19A236BA+108C+8A3+12−12B3−3B2A2+54BAC+81C2+12CA313

(1.8)

Since the expression of Eigen values are very complicated and difficult to get some concrete information regarding the nature of Eigen values. Therefore to simplify the analysis, here we choose some specific values of a, b and c.

Now substituting the value of x^*, z^* from equation (1.5) and setting the values of a=b=0.2 and c=5.7 we get A=11.55, B= -29.25 and C=81.6

This equation (1.6-1.8) become

$\boldsymbol{\lambda}_1$= 0.0971-0.9957 i

$\boldsymbol{\lambda}_2$= 0.0971+0.9957i

$\boldsymbol{\lambda}_3$= −5.68718

Now we see that $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2$ are complex conjugates and real part of $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2$ are positive and $\boldsymbol{\lambda}_3$ is negative. Thus equilibrium point will be unstable.

Similarly one can also choose other combinations of a, b and c and obtain the stability of equilibrium points for such values.

For the second set of fixed points we follow the same procedure to find the Eigen values. Thus again fixing a=0.2, b=0.2 and c=5.7

we get A=1.45, B=1.87 and C=5.8 and Eigen values are

$\boldsymbol{\lambda}_1 = -.5107 - 1.1976\, i$

$\boldsymbol{\lambda}_2 = -.5107 + 1.1976i$

$\boldsymbol{\lambda}_3 = 3.42152$

Again we see that $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2$ are complex conjugates and real part of $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2$ are negative and $\boldsymbol{\lambda}_3$ is positive. Thus equilibrium point will be unstable.

1.1.3 Eigen Vectors

The stability of each of these fixed points can be analyzed by determining their respective Eigen values and eigenvectors[10]. For the first set of eigen value equation (1.5) are computed as

$$V_1=\begin{pmatrix} 0.7073 \\ -0.07278-0.7032i \\ 0.0042-0.0007i \end{pmatrix}$$

$$V_2=\begin{pmatrix} 0.7073 \\ -0.07278+0.7032i \\ 0.0042+0.0007i \end{pmatrix}$$

$$V_3=\begin{pmatrix} 0.1682 \\ -0.0286 \\ 0.9853 \end{pmatrix}$$

These eigenvectors have several interesting implications. First, the two Eigen value/Eigen vector pairs (V_1 and V_2) are responsible for steady outward slide that occurs in the main disk of attractor. The last Eigen value/Eigenvector pair is attracting along an axis that runs through the center of the manifold and accounts for z motion that occurs within the attractor shown in Fig. (1.4)

The blue line corresponds to the standard Rossler attractor generated with a=0.2, b=0.2, and c=5.7.

The red dot in the center of this attractor is FP_1. The red line intersecting that fixed point is an illustration of the repulsion plane generated by v_1 and v_1.

The green line is an illustration of attracting v_3. The magenta line is generated by stepping backwards through time from a point on the attracting eigenvector which is slightly above FP_1 it illustrates the behavior of points that becomes completely dominated by that vector. Note that magenta line nearly touches the plane of attractor before being pulled upward into the fixed point; this suggests that the general appearance and behavior of the Rossler attractor is largely a product of the intersection between the attracting v_3 and the repelling V_1 and V_2 plane. Specifically it implies that a sequence generated from Rossler equations will begin to loop around FP_1, start being pulled upwards into the V_3 vector, creating the upward arm of a curve that bends slightly inward toward the vector before being pushed outward again as it pulled back towards the repelling plane.

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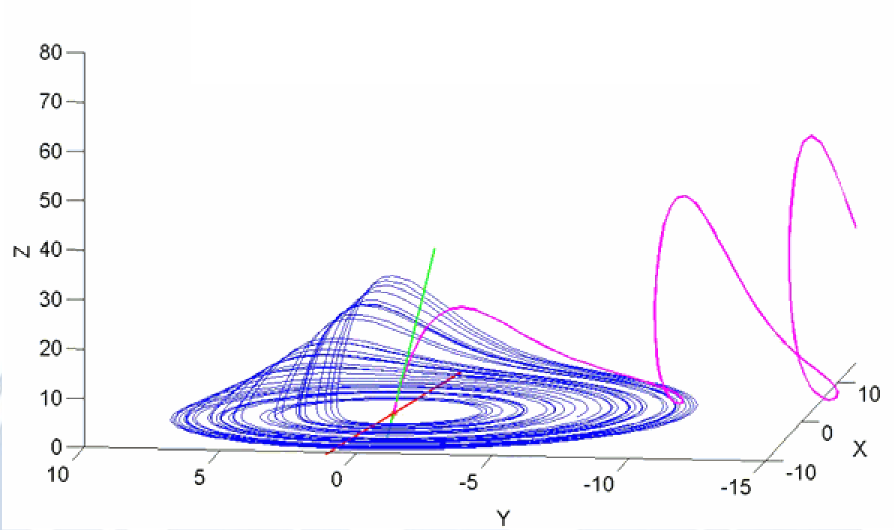

Fig. (1.4) Examination of central fixed point eigen vectors. The blue line corresponds to the standard Rossler attractor generated with a=0.2, b=0.2, and c=5.7.

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