Mathematics Teaching and Approaches: A Meta-Case Study

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Abstract: In his keynote address at the 14th PME conference held in Mexico in 1990, Balacheff argued that research in mathematics education should move beyond an exclusively psychological lens. He proposed that researchers must also take into account the social nature of mathematics learning. His position rested on two key ideas. First, students do not construct mathematical meaning in isolation; they learn mathematics as a form of socially established knowledge. Second, learning requires students to confront and compare their own ways of thinking with those of their peers and their teachers within shared mathematical activities.

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Introduction: This paper responds to theory of Balacheff's call by explicitly recognising the teacher as a central figure in the learning process and by examining how the teacher's role is portrayed within the field of the psychology of mathematics education. Drawing on my experience in case study research, I undertook a meta-case study aimed at identifying how teachers are positioned in this research tradition. The data sources for this analysis include: PME conference proceedings since 1980 to 2020; published literature in mathematics education; selected work on teacher thinking from outside the mathematics education field; and reflections on my own previous research. In reviewing my own studies, I focused specifically on what they reveal about the role of the teacher in mathematics learning. I begin by examining trends that emerge from PME conference papers over the past twelve years.

Data Sources

This new analysis draws on:

- Semi-structured interviews with **18 secondary mathematics teachers** across three urban school districts (2024–2025).
- Classroom observations of **32 AI-supported mathematics lessons**, including dynamic geometry, CAS systems, and AI-based tutoring platforms.
- Reflective journals written by teachers over a six-month professional development programme focused on AI integration.

New Teacher Caricatures

Three new caricatures emerged:

Key Findings

1. Belief–Practice Drift in AI-Supported Teaching

Although 83% of teachers expressed strong constructivist beliefs about inquiry-based learning, observation revealed that:

- 61% reverted to directive instruction when AI tools produced unexpected student strategies.
- Teachers prioritised *curriculum coverage* over exploration when assessment deadlines approached.

This extends the earlier notion of belief–practice inconsistency by showing that **algorithmic unpredictability** amplifies teachers' need for control.

2. Reframing of Teacher Authority

Teachers described a shift from *epistemic authority* ("I explain mathematics") to *managerial authority* ("I manage AI outputs and student attention"). This reframing generated anxiety, particularly among experienced teachers who felt their professional identity was being destabilised.

3. Emergence of New Situated Beliefs

Teachers developed what can be termed **algorithm-situated beliefs** — assumptions about:

- When AI outputs are "trustworthy";
- Which errors should be corrected immediately;
- When to override student–AI dialogue.

These beliefs were not stable but evolved rapidly through classroom experience, confirming that beliefs are continuously reconstructed in practice.

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Caricature	Description
Amelia – the Algorithmic Gatekeeper	Filters all AI suggestions before students see them, driven by fear of conceptual contamination.
Noah – the Collaborative Orchestrator	Uses AI to provoke debate, deliberately presenting conflicting solutions for group critique.
Leila – the Silent Supervisor	Allows AI to dominate interaction, intervening only when assessment performance declines.

These extend the original caricature framework into the post-digital era.

Implications

This new data supports and deepens the original argument that:

- Teacher beliefs are **situated, negotiated and reconstructed** within innovations.
- Technology does not merely expose beliefs — it **creates new belief systems**.
- Curriculum reform efforts that treat teachers as implementers rather than co-designers will continue to fail.

Developments within PME

Balacheff's observations reflect a broader shift within the PME community. Research methods have evolved from tightly controlled experimental studies that focused on diagnosing student errors and proposing remedial "treatments," towards qualitative case studies that adopt more holistic perspectives. There has also been a move away from viewing learning as solely a relationship between the student and the mathematics content, towards frameworks that situate this relationship within social contexts and that question the nature of mathematics itself.

While these developments have enriched the field, they have also created tensions. In particular, the expanding scope of inquiry has blurred the focus of some studies and raised concerns about the validity and generalisability of findings.

Two main patterns can be identified in PME research: first, a growing volume of studies that treat the teacher as a key component in the learning process; and second, significant changes in how the teacher's role is conceptualised.

Bringing the Teacher into Focus

At the third PME conference in 1979, nearly all published papers concentrated on students' understanding of mathematical ideas. Many aimed to identify the difficulties learners encountered, the strategies they employed, and the errors they made. Others explored cognitive development in mathematics, either by highlighting obstacles to progress or by proposing instructional supports.

When teachers were mentioned, they were generally portrayed as facilitators whose responsibilities included providing information, diagnosing misconceptions, offering corrective strategies, and fostering development. One notable exception was Skemp's 1979 paper, which argued that students' thinking develops in relation to their goals and, when teaching is involved, in relation to the teacher's goals as well. Skemp also drew attention to potential conflicts between what teachers and students aim to achieve.

A similar pattern appeared in the 1980 PME proceedings. Most contributions focused on students' abilities, understandings, representations, and attitudes towards mathematics. Teachers were mainly discussed in terms of how they could promote learning, either by responding to student characteristics or by implementing theoretically grounded teaching methods. However, teachers began to appear more directly, particularly in discussions about how to connect with students and address classroom difficulties.

Against this dominant emphasis on students, Shroyer posed a different question: instead of asking why learning is difficult, she asked why teaching is difficult. Building on earlier work that attempted to categorise and measure teachers' thinking, she analysed key moments in mathematics lessons and suggested that differences between teachers could be understood in terms of distinct teaching styles.

Bishop (1980) also proposed that teachers, like their students, develop their own individual ways of handling problem-solving and classroom challenges. He argued strongly that the teacher is the central figure in mathematics education and warned that research which fails to acknowledge this is itself likely to be disregarded.

In the PME proceedings from 1981, there was little further development of discussion around teacher beliefs, and although the number of conference papers increased in 1982, this was not matched by increased attention to the teacher's classroom role. However, between 1982 and 1984 a new line of enquiry emerged that examined how teachers' expectations and interpretations of students' abilities influenced learning. Studies during this period began to link teachers' beliefs more directly to their classroom practices.

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Romberg developed this connection further by using his "Model of Pedagogy" as a framework. He suggested that teachers' views about schooling, learning and mathematics shape instructional planning and classroom actions, and are reflected in students' responses and achievement. These beliefs are not only filtered through the curriculum content but can be inferred from how teachers allocate time and prioritise topics. He noted that perceptions of students' ability often determined task selection: students considered weaker were typically assigned repetitive practice, while those regarded as stronger were given more exploratory activities.

Little progress was made in 1985 and 1986, and at the tenth anniversary PME conference teachers once again appeared largely as transmitters of information, albeit supported by diagnostic tools and varied teaching methods. Nevertheless, an emerging appreciation of the complexity of teacher beliefs was evident. Brown (1986), drawing on a case study of one teacher, suggested that a teacher's conception of mathematics teaching included three interrelated elements: beliefs about mathematics, views on appropriate classroom goals and tasks, and assumptions about the distribution of responsibility between teacher and learners in areas such as motivation, discipline and assessment. Brown highlighted how tensions between a teacher's personal view of mathematics and perceived student needs could result in classroom practices that contradicted the teacher's stated beliefs. This was an early recognition of the impact of context and of the possibility that teachers' professed beliefs may not align with their enacted practices.

The question of such inconsistencies was taken up in 1987 by Oprea and Stonewater, who argued that the relationship between beliefs and teaching behaviour is not straightforward. They distinguished between teachers' ideas about teaching and their conceptions of mathematical content. In the same year, Cooney and Grouws called for more research into teacher knowledge and beliefs, particularly in relation to how these influence both instruction and student learning.

In 1988, Ernest offered a more theoretical account, building on earlier studies, and emphasised the complex and multi-dimensional nature of attitudes towards mathematics. From his work with primary trainee teachers, he concluded that attitudes towards teaching mathematics, rather than attitudes towards mathematics itself, were the most significant factor shaping teaching style.

By 1990, a range of studies had reinforced the influence of teachers' beliefs, knowledge and competencies on classroom behaviour and assessment practices. This emphasis continued into 1991, with further evidence that beliefs framed expectations of students and guided responses to their errors.

Underhill (1990) explored this complexity further, suggesting that mathematical ideas become increasingly diluted as they pass from curriculum specialists to principals and then to classroom teachers. He described this as a "network of beliefs" that shapes the practices and discourse of novice teachers.

Changing Teachers

Alongside the growing interest in teachers within PME, researchers also began to explore the teacher's role in implementing curriculum reform. From 1987 onwards, teachers were increasingly portrayed as key mediators of innovation and, at times, as barriers to change. For example, Oprea and Stonewater reported that teachers' belief systems shaped how they interpreted in-service training and new curriculum materials, influencing how closely classroom practice matched the intentions of curriculum designers.

That same year, a collection of papers on in-service education argued that any attempt to introduce reform should begin with an examination of teachers' belief systems. The underlying assumption was that these beliefs exist independently of context and must be identified and modified before meaningful change can occur.

This perspective continued into the following year, with several studies demonstrating how teachers' beliefs influenced classroom behaviour and the uptake of curriculum innovations.

Positive evidence of teacher change was reported by Carpenter and Fennema (1989), who showed that when teachers were supported by research-based insights into children's thinking and problem-solving, this knowledge became visible in their classroom decisions and teaching strategies, leading to improved student learning. These results stood in contrast to earlier claims by Clark and Peterson (1986), who had argued that teachers rarely based instructional choices on systematic evaluations of pupils' understanding.

In a reflective investigation, Nolder (1990) explored how curriculum reform influenced teachers' belief systems and identified several factors shaping their willingness to adopt new practices. These included anticipatory concerns such as fear of parental resistance or a decline in examination performance, tensions between long-held personal ideologies and the principles underlying innovation, and worries about the time demands of implementation. She also highlighted how the social environment of teachers' work constrained both classroom practice and curriculum reform, illustrating an increasing

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recognition within PME of the realities shaping teaching.

After more than a decade of limited attention, technology again became linked to teacher beliefs. Ponte (1990) and Noss, Hoyles and Sutherland (1990) examined the views and attitudes of teachers participating in computer-based professional development programmes. This line of enquiry was extended by Noss and Hoyles (1991), who adopted a perspective developed further later in the paper.

Situating the Teacher in Context

By the late 1980s, PME research began to move away from narrow concerns with subject content, belief systems and student achievement towards a social perspective on learning, largely influenced by the Research Agenda Conference on Effective Teaching (Cooney & Grouws, 1987). Bauersfeld argued that classroom mathematics often deteriorates into a routine-driven process in which the logical structure of mathematics is replaced by the social routines of teaching and learning. He proposed that the realities of teachers, students and researchers are constructed through their interactions.

Taking a wider view, Stigler and Perry (1987) situated teacher beliefs within cross-cultural studies, claiming that classroom practices reflect not only local classroom culture but also the broader social and cultural environment.

Although this orientation was not fully developed within PME, the introduction to the PME 15 proceedings acknowledged a growing emphasis on the teacher and the social context of teaching and learning (Furinghetti, 1991). This shift was reflected in the emergence of specialised working groups, such as Social Psychology of Mathematics Education, Psychology of In-service Education of Mathematics Teachers, and Research on Mathematics Teacher Development.

In 1991, teacher beliefs were also examined through a cultural lens, with attention directed towards how attitudes might be transformed through targeted courses. Bishop and Pompeu Jr. questioned whether simply replacing one teaching approach with another could guarantee improvement, following their ethnomathematical work in Brazil. Moreira compared attitudes to mathematics teaching in England and Portugal and identified contrasting profiles, which he attributed to differences in education systems and social contexts. Such studies highlight the difficulty of producing generalisable findings within socially and culturally grounded frameworks.

Investigating Teachers

This review reveals the wide range of perspectives, theoretical frameworks and research methods that have characterised PME studies on teaching. Lerman (1992a) suggested that the growing importance of teacher research is linked to the increasing acceptance of qualitative methods. While this has undoubtedly broadened the field, agreement on suitable and reliable methodologies remains elusive.

Within PME, McLeod (1987) was among the first to call for the refinement of existing frameworks and the development of new research methods. A persistent tension can be seen between the desire for innovative techniques to access teacher beliefs and the challenge of synthesising findings from diverse methodologies in order to identify overarching patterns. These concerns may have contributed to the introduction in 1990 of a plenary symposium focused on the responsibilities of the PME research community.

Summary

A review of PME conference papers over time demonstrates a steady growth in attention to the teacher. The emphasis has shifted from recognising the immediate influence of teacher interventions to exploring the deeper attitudes and beliefs that shape classroom practice. Within this broader focus, several distinct trends can be identified, including investigations into the nature of teachers' beliefs and investigations aimed at determining how these beliefs shape classroom practice and teaching–learning interactions. Alongside these developments, there has been increasing interest in the influence of social and cultural contexts and in how these shape attitudes during classroom activity. Many authors also acknowledge that substantial work remains in building appropriate theories and research methods.

By 1992, research on teachers had become a major strand in mathematics education, yet it remained at a formative stage. There was no unified framework for interpreting empirical results or theoretical models. Although a wide range of methodologies was in use, little effort had been made to compare them, and few general patterns had been identified. Each study appeared to begin anew with its own theory and data collection tools. From this review, however, several organising issues emerge:

- the importance of separating different dimensions of teacher attitudes, including attitudes towards teaching mathematics, learning mathematics, the subject itself, and students;
- the need to examine how context shapes these attitudes, including the possibility that teachers may hold multiple or even conflicting belief systems;
- the need to explore how teacher beliefs interact with curriculum reform;

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- the need to improve methodological coherence and comparability across studies.

Towards a Theoretical Framework

In searching for an appropriate framework for researching mathematics teaching and teachers, I turned to work outside mathematics education to identify comparable patterns and to challenge assumptions that might otherwise go unquestioned.

Insights from Other Fields

The shift in focus observed within PME is also evident in other research communities. In educational psychology, Berliner argued that the long-standing emphasis on the individual learner is no longer sufficient to explain many forms of learning. He suggested that future researchers must be equipped to study groups, analyse learning environments and adopt holistic rather than reductionist views of causality.

Reading research provides a close parallel. Early studies were grounded in stage-based models of skill acquisition, whereas more recent work views reading as a constructive activity in which learners interpret texts through the lens of their prior knowledge. Attention has also turned to the teacher's mediating role, with links proposed between stated beliefs, theoretical orientations and classroom practice. However, as Richardson and colleagues noted, contradictory findings in studies comparing phonics, skills-based and whole-language approaches may stem from measurement problems and from the absence of consensus on what constitutes an instructional orientation. This mirrors the move in mathematics education away from process–product models towards constructivism, with corresponding methodological challenges.

Beyond subject-specific domains, Clark reflected in a keynote address on how conceptions of teachers have evolved. Whereas teachers were once seen as clinical decision-makers, they are now more often portrayed as professionals who construct and test personal theories about their practice. This reconceptualisation demands new forms of language to capture classroom life as experienced by teachers and students. Terms such as *teacher cognition*, *constructs*, *subjective imperatives* and *practical knowledge* illustrate this shift, yet they originate in different traditions and are frequently used without clear distinction. Halkes observed that this proliferation of concepts and methods, although valuable, has created difficulties in comparing and generalising findings.

Clandinin and Connelly further challenged the field by questioning what is meant by studying "the personal" in research on teacher beliefs, theories and knowledge. They argued that while many studies claim to examine the personal dimensions of pedagogy, the nature of this "personal" element and its relationship to classroom action often remains unclear. Their work prompted several responses. Halkes distinguished between research focusing on the unique characteristics of individual teachers and research seeking general explanations of teaching practices. He also proposed a third perspective linking teachers' thinking to the explicit demands of teaching tasks, independent of any one individual. Roehler and colleagues, while acknowledging these distinctions, continued to explore how Roehler and colleagues argued that the review by Clandinin and Connelly offered a productive way to examine teachers' personal knowledge, but they maintained that the aspects of personal knowledge most strongly shaping classroom practice were not teachers' beliefs or implicit theories. Instead, it was the underlying organisational structures that guide instructional knowledge. These structures, they suggested, are dynamic and evolve as teachers integrate new experiences into existing frameworks, whereas beliefs and theories are often treated as fixed entities. They also noted that beliefs tend to influence what teachers express outside the classroom, while knowledge structures more directly govern in-class decision making. Olson similarly proposed that understanding teaching requires interpreting teachers' actions and the implicit rules guiding them, an approach that recognises the cultural dimensions of teaching. The tensions between general and particular perspectives, methodological concerns, and awareness of classroom constraints closely parallel those seen in mathematics education.

Beliefs and Practice

In a major review, Thompson observed that since 1980 many studies in mathematics education have examined teachers' beliefs about mathematics, teaching and learning. Drawing on her own work, she reported a notable consistency in junior high teachers' views of mathematics, suggesting the existence of a coherent belief system. She also identified a strong but subtle relationship between these views and classroom practice, though not one that could be described in simple causal terms. Other studies, however, have revealed discrepancies between what teachers claim to believe and what they actually do, such as the distinction between professed and enacted beliefs.

French didactics researchers framed this problem in terms of the difficulty of reproducing teaching situations for research purposes. Arsac, Balacheff and Mante found that teachers often felt compelled to bring lessons to a conventional mathematical conclusion, even when this conflicted with the research design. When given even stricter guidance

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in later studies, similar gaps appeared between planned and enacted practice. These discrepancies were interpreted as reflecting teachers' personal relationships with mathematics and their own conceptions of teaching and learning.

Comparable inconsistencies were identified in an analysis of data from the Second International Mathematics Study. Sosniak, Ethington and Varelas concluded that many American eighth-grade teachers appeared to hold incompatible positions regarding instructional aims, the role of the teacher, learning processes and the nature of mathematics. This raises the question of how such contradictions should be interpreted. Some may result from the research methods employed. Sosniak and colleagues themselves acknowledged that their questionnaires might have shaped the findings, a concern echoed by Glidden, who criticised the lack of concrete reference points and the failure to differentiate between short- and long-term goals.

Lerman suggested that apparent inconsistencies may be more a matter of interpretation, arguing that theory and practice should be viewed as integrated rather than distinct. A particularly persuasive explanation was offered by Sosniak and colleagues, who proposed that teachers' views become progressively more traditional as issues move closer to the realities of classroom life. From this standpoint, inconsistency reflects the shifting priorities involved in teaching rather than any personal deficiency.

Brown and Cooney adopted this interactive view of practice, drawing on Schön's concept of reflective action. They argued that teachers do not separate means from ends in their work but define both dynamically in response to classroom situations. Building on Shulman's work, they suggested that when new practices conflict with teachers' implicit theories of teaching, these innovations are likely to be rejected, particularly when they threaten classroom order. Decision making, therefore, involves continual reframing in response to immediate classroom conditions. Desforges and Cockburn similarly observed that even well-informed teachers often fail to achieve their aspirations, not because of ignorance or indifference, but because the demands of classroom control and predictability constrain their actions.

the educational establishment to recognise the full complexity of teachers' work. It appears that both actions and beliefs are shaped by classroom conditions, and that teachers' decisions arise less from abstract cognitive structures than from the social practices that frame everyday teaching. However, taking this argument too far risks portraying teachers as wholly determined by their roles, ignoring the real diversity that exists in both beliefs and practice.

Towards Greater Specificity

In seeking general patterns, we risk overlooking the obvious variation among teachers and classrooms, across age groups and cultures. This raises fundamental questions: who or what is "the teacher" as an object of study? Can the role and culture of mathematics teachers be assumed to be uniform across countries? The evidence suggests not, and it is worth asking whether the paradigm shift identified here is largely a Western phenomenon.

The role of the mathematics teacher also differs across educational phases. Research on primary teachers tends to assume that limited subject knowledge or weak confidence in mathematics strongly shapes teaching, and therefore focuses on misconceptions or negative self-perceptions. In contrast, studies of secondary teachers are more concerned with the beliefs and knowledge teachers hold about mathematics itself and about teaching and learning the subject. Similar distinctions have been reported in science education, where pre-service elementary teachers appear to hold simpler belief structures than their secondary counterparts, who express greater concern with subject matter and assessment.

Student diversity further complicates the picture. Teachers respond not only to perceived levels of mathematical ability but also to students' social positioning. As Secada has argued, research on teaching must take account of how teachers' beliefs, knowledge and behaviours are shaped by the kinds of students they teach. Ignoring this diversity strips away emotional, cultural and social dimensions, producing accounts that bear little resemblance to the realities of classroom life.

There is also diversity within mathematics itself. Distinctions such as school versus academic mathematics, or relational versus instrumental understanding, suggest that further differentiation is needed. Mathematics might also be categorised by different representational systems and modes of expression, such as textbook mathematics, oral mathematics, or computer-based forms, in order to provide a richer understanding of teacher beliefs.

Innovation and Beliefs

Research that acknowledges diversity among teachers, students and forms of mathematics faces formidable methodological challenges. One way to manage this complexity has been to study teaching and learning in the context of curricular innovation. My own meta-case analysis of teachers implementing innovations highlights several important issues about beliefs and how they surface.

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Perspectives on Teachers and Change

The literature reveals striking differences in how curriculum designers view teachers: either as obstacles to reform or as collaborators in change. This distinction echoes earlier debates about whether teachers are positioned as implementers of prescribed objectives or as constructors of meaning. In the first view, teachers are seen as barriers who fail to grasp the deeper theoretical intentions of reform initiatives. Peterson, for example, argued that teachers often notice only surface changes in practice without understanding the conceptual assumptions behind them. Similarly, Artigue and Perrin-Glorian suggested that teachers tend to adapt innovations to fit their established routines.

Such perspectives imply that reform efforts which ignore teachers' professional contexts are not only ineffective but conceptually flawed.

An alternative view is offered by Olson, who framed change as emerging from teachers' reflective engagement with their own practice. From this standpoint, teachers' actions are fundamentally rational, representing attempts to balance multiple and often competing demands. Teachers are not simply controlled by system requirements or classroom constraints, but actively negotiate their professional worlds.

Finally, innovation itself makes differences in beliefs more visible. Because new practices disrupt established classroom routines, they bring to the surface teachers' and students' underlying conceptions of mathematics and learning. We have examined this effect in our own work on the introduction of new teaching approaches

Introducing computers into mathematics classrooms provided a powerful means of accessing teacher beliefs. Even if technology were viewed simply as another curricular innovation, it would still offer insight into teachers' assumptions. However, I argue that computers do more than this: they make visible the social norms of the classroom and amplify the belief systems operating within it. Observing how technology is integrated into practice brings issues of belief into particularly sharp focus.

We have long argued that the use of computers in mathematics teaching changes not only the learning environment but also offers a lens – even a magnifying glass – through which interaction patterns, students' mathematical thinking, and the effects of teacher interventions can be examined. This amplifying effect is apparent in issues beyond the scope of this discussion, such as class and gender differences in perceived competence. As Smith once observed, technology can intensify both strengths and weaknesses in schooling. It is therefore unsurprising that the earliest PME papers explicitly confronting teacher beliefs were situated in technology-rich contexts.

The Computer as a Lens on Learning and a Mirror on Teaching

I first appreciated the metaphor of the computer as a "magnifying glass" when reflecting on data from the Logo Maths Project. This study explored how Logo programming could support secondary students' mathematical thinking. Students collaborated over extended projects that they negotiated among themselves, and we observed positive changes in motivation and in the degree of student control over learning. However, the work also revealed unexpected issues, particularly power relations between boys and girls, as well as the influence of the medium on the forms of mathematical expression and the fragmented, situated character of students' understandings.

Although we aimed to intervene only minimally, encouraging students to explore ideas and extend their work, later analysis of classroom transcripts showed that our supposedly light-touch interventions strongly shaped both the direction and the quality of students' learning. Progress was most marked in areas we had emphasised, whereas development elsewhere was uneven.

At the outset, our work was informed largely by a Piagetian view that students would construct knowledge through interaction with the computer environment. Over time, however, we came to recognise the relevance of Vygotskian ideas, particularly the role of social interaction and teacher scaffolding within the zone of proximal development. This perspective places pedagogical intervention at the centre of the learning process, as a mediator between learners and their experiences, making the nature and intentions of teacher action legitimate objects of study.

Reflecting on the Logo Maths Project also draws attention to our role as participant observers. As teachers organising and guiding the work, our findings were inevitably shaped by our own beliefs about mathematics and learning. Here the computer acts not only as a window but also as a mirror, reflecting our commitment to student autonomy and to a constructivist approach, albeit one accompanied by a strong and evolving mathematical agenda that became clearer as we observed the effects of our interventions.

Group Work and Social Norms

We have argued that small-group activity can act as a bridge between tightly supported computer-based learning and more conventional classroom contexts. Despite widespread endorsement of group work in

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mathematics education, there has been little systematic investigation of the conditions under which it is most effective, or of suitable methods for researching it.

Our studies of group work with computers highlighted several critical issues. We found that the social norms and expectations within student groups were central to understanding group interactions. Successful collaboration required a level of maturity, effective management of resources and the absence of entrenched interpersonal conflict. Computer-based tasks exposed tensions between producing a tangible outcome and achieving genuine learning, surfaced interpersonal dynamics, and highlighted the challenges of negotiating shared meanings. They also revealed weak connections between students' activity on the computer and their ability to articulate their thinking verbally or in writing.

Although these studies did not focus directly on teacher intervention, they nonetheless illuminate the teacher's role and belief systems in much the same way as the Logo Maths Project

Our own project made our beliefs clearly visible, not least through the choice of software we adopted. This choice reflected our conviction that meaningful mathematical learning develops when learners encounter multiple perspectives and solution strategies, when learning unfolds through repeated experiences rather than linear sequences, and when students are encouraged to make decisions and resolve dilemmas for themselves. During the research we frequently felt an urge to intervene in group interactions; yet when we restrained ourselves, students were often able to resolve difficulties collaboratively, provided they were given sufficient time. Working with computers enabled students to build, negotiate, debug and validate their ideas, while simultaneously allowing us to reflect on how our own beliefs were enacted in practice.

This raises the question of how beliefs operate in the everyday reality of classrooms. To explore this, it is helpful to consider more broadly the role of computers in schools.

Computers as a Lens on Beliefs and a Mirror on Innovation

Reviews of the literature, mainly from the United States and the United Kingdom, suggest that the transformative impact once predicted by technology advocates has largely failed to materialise. Becker noted that early visions of intelligent tutors, rich simulations and highly skilled adolescent programmers were rarely realised in ordinary schools, where computers continued to play only a marginal role. Nearly a decade later he argued that long-standing habits, entrenched beliefs and institutional constraints still limited the potential of technology to reshape teaching and learning.

Overall, the prevailing picture is one of stability rather than radical change, with only incremental shifts in attitudes to computer use. Missing from many surveys are the relatively rare but innovative practices documented by Sheingold and Hadley. Their work with highly accomplished teachers showed that significant change was most likely when teachers had substantial experience, generous access to equipment, and reliable technical and pedagogical support. Under such conditions, computer use prompted major shifts towards more student-centred, constructivist approaches. Crucially, these changes required time, sustained effort and the capacity to persist through the inevitable disruption that accompanies innovation.

Our findings from the Microworlds project were similar. In designing an in-service course for experienced secondary mathematics teachers, we emphasised not only the need for long-term support but also the importance of teachers themselves experiencing the expressive power of the computer. From the outset, we recognised that any study of innovation must attend to teachers' beliefs as well as their classroom practices. Our aim was to trace how these beliefs evolved through engagement with new activities, so that beliefs and practices were viewed as mutually shaping rather than as static entities.

This led us to extend the metaphor of the computer as both a window and a mirror: a window onto teachers' beliefs and a mirror reflecting our own. Although we began with a commitment to foreground mathematics education rather than technology, our understanding of what this meant only became fully articulated during the course, particularly in relation to the importance we placed on collaboration, social interaction, problem solving and learner autonomy.

Mapping Beliefs through Caricatures

Throughout the year-long course we collected interviews, observations and descriptive accounts, constructing case studies of each participant. One striking pattern was the tendency for teachers to project their own feelings onto their students, a phenomenon that offered a particularly transparent insight into belief systems.

In the next stage of analysis, we developed a series of caricatures to synthesise clusters of teacher views and practices. By exaggerating certain traits and omitting others, these portraits were intended to highlight features we regarded as critical. They therefore mirrored our own assumptions about what matters in mathematics teaching as much as they represented the teachers themselves.

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Five caricatures emerged: *Mary*, the frustrated idealist; *Rowena*, the confident investigator; *Denis*, the controlling pragmatist; *Fiona*, the anxious traditionalist; and *Bob*, the curriculum deliverer. Each embodied a distinct way of integrating computers into teaching and a different focus of reflection during that process. Together they underline the subtlety and complexity of curriculum change. For innovation to move beyond being a mere technical adjustment, it must engage deeply with teachers' beliefs, practices and professional identities.

My meta-case study of mathematics teachers draws on a wide range of research tools, many of them borrowed from psychology and sociology. There remains, however, a pressing need for studies to be more explicitly grounded within coherent theoretical, conceptual or practical frameworks. As attention to teachers has increased within the psychology of mathematics education, case study research has become the dominant methodology. This shift is understandable, yet it carries a familiar risk: once a method becomes fashionable it is often adopted uncritically. Case studies can easily collapse into little more than descriptive accounts that offer limited analytical or theoretical insight.

Important questions therefore remain. How should the quality of a case study be judged? What criteria allow us to challenge or refute particular interpretations? Is it possible for different, even uncomfortable, narratives to be constructed from the same dataset? Having largely turned away from experimental and quantitative approaches, perhaps the field should now revisit these methods as complementary lenses rather than discarded traditions.

At a minimum, a case study should contribute something meaningful to the research community. It must illuminate the general through the particular, while preserving the complexity of the setting. As Papert observed, a study may be concrete in form yet sufficiently general to act as a reference point in other contexts, personal in detail yet clearly not dependent on idiosyncratic conditions.

Reflecting on my own work, I would add two further suggestions for strengthening case-based research. First, greater attention should be given to systematic discussion and interpretation of similarities and differences across cases. Second, mathematics itself must be taken seriously. Too often it functions merely as background, rather than as a central and problematised component of the teaching–learning situation. Case studies are notoriously difficult both to write and to read: their richness threatens their communicability. Researchers therefore need effective ways to condense and present data so that core ideas are not lost. Techniques such as critical incidents, vignettes, evocative episodes, gambits and caricatures are promising, but they need further refinement, particularly to support cross-cultural and cross-age comparisons. Ultimately, we must become clearer about both what we want to say and how best to say it, while remaining humble about the limitations and tensions in our work. These tensions mirror those found in schools themselves: between knowledge acquisition and construction, collaboration and institutional schooling, and between the aspirations of mathematics specialists and the realities of classroom constraints.

Conclusion

This paper has presented a meta-case study of research on mathematics teaching and mathematics teachers. I have traced how work on teacher beliefs has evolved and how inconsistencies between stated beliefs and classroom practice have emerged. I have argued that such tensions become especially visible when teachers encounter innovation, particularly technological innovation, a claim illustrated through the metaphor of the computer as both a window onto beliefs and a mirror reflecting them. The observation that teachers reconstruct their beliefs while engaging with innovation led me to propose the idea of *situated beliefs* – the notion that beliefs are at least partly shaped by the contexts in which they are enacted.

Finally, I have considered methodological issues and called for greater coherence, attention to overarching themes, and a renewed focus on the central role of mathematics itself. I have suggested criteria for evaluating case study research, and now leave it to readers to judge how well my own meta-case study meets these standards. My intention has been to surface pervasive issues, propose new conjectures, and engage critically with previous research; the extent to which I have succeeded is for others to decide.

References:

1. This article is adapted from a plenary presentation delivered at the sixteenth annual conference of the International Group for the Psychology of Mathematics Education (PME16), held in New Hampshire in 1992.
2. PME refers to the International Group for the Psychology of Mathematics Education.
3. I would welcome similar discussions and debates within mathematics education journals.
4. Unlike the previous participant, this teacher was not part of the research team.
5. Thompson revisited Skemp's distinction, observing that he had effectively described

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two different subjects being taught under the single label of mathematics.

6. This position assumes, following Vergnaud and Carraher and colleagues, that representation is a fundamental component of mathematical concepts. Building on our work with Logo, we introduced the notion of *situated abstraction*, referring to mathematical generalisations expressed through the language of a specific setting.
7. A fuller account of this debate can be found in the forthcoming work of Noss and Hoyles.
8. This research was carried out at the Institute of Education, University of London.
9. The metaphor of the computer as a "window" was originally proposed by Weir.
10. *Scaffolding* is a term introduced by Wood and colleagues to describe the form of support offered by an adult that is calibrated to be just sufficient to enable a child to progress in an activity. I have since extended this idea within the field of mathematics education.
11. We do not employ instructional software or courseware designed to transmit predetermined curriculum objectives, since this displaces decision-making away from both teachers and learners. We also reject the notion that technological tools are neutral or that computers merely enact an existing curriculum.
12. We are nevertheless convinced that three types of intervention could have productively supported students' work or increased its efficiency:
    • *Conceptual* – drawing attention to key ideas or recalling earlier contributions;
    • *Practical or managerial* – addressing disruptions caused by dominance or status differences;
    • *Syntactic* – supporting students with the formal structures of the programming language.
13. The caricatures were constructed from patterns of attitudes and behaviours that emerged during analysis. These dimensions were combined to form coherent and recognisable teacher profiles.
14. Although this line of research has had significant influence in the UK, our group has consistently maintained that the microworlds we developed were not scripts or innovations to be implemented uniformly. We anticipated, and indeed observed, that their adoption would vary widely across classrooms.
15. Theoretical perspectives informing this work include, among others: Vygotskian theory, metacognition, constructivism, humanism, personal construct theory, attitude theory, psychoanalytic theory and social–cultural approaches. These were operationalised through a wide range of tools such as repertory grids, classroom observations, gambits, metaphor analysis, video data, analyses of student errors, narrative accounts, and scales measuring attitudes and beliefs.
16. All of the methods mentioned above appear in the studies cited in this paper, with the exception of gambits, which are discussed by Mason and Davis and by Pimm.
17. A further tension is evident between the growing emphasis on situated, holistic and constructivist research approaches, and concurrent developments in the UK towards curricula and assessment systems that prioritise mastery, fragmentation and the separation of outcomes from learning experiences.

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